APPENDIX C

PROBABILISTIC LEVEE FAILURE METHODOLOGY

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The mathematical models used in the calculation of the probability of levee failures are described in this Appendix. To apply the probabilistic approach, we need to first parameterize the point estimates of the fragilities.

C1.   PARAMETRIC MODELS FOR LEVEE FRAGILITIES

The point estimates of the levee fragilities developed for this study were fit to simple equations to facilitate the probabilistic calculations. The simplified models for the median and coefficient of variation (cov) for both liquefaction and non-liquefaction induced failures are given below.

Fragility Curves for Liquefaction Induced Failures

The median fragility liquefaction for ln liquefaction induced failures is modeled by

$$frag_{Li}(pga,M) = 0.8 exp(p_1 + p_2[ln(pga) + c_1 + c_2 M + c_3 M^2 + c_4 M^3] + c_{5i})$$

The coefficients p_1, p_2, c_1, c_2, c_3, c_4, and c_5 were estimated from the central value of the range given in the point estimates. The 0.8 factor represents the interpretation of the sub-team that the median fragility is not at the center of the range given in the point estimates, but rather it is approximately at 40% of the range.

The coefficient of variation for all zones is modeled by

$$cov_L = (b_1 + b_2 pga)/1.3$$

with a constraint that it not be less then 0.3/1.3. The factor of 1.3 represents the interpretation of the sub-team that the range on the fragility given in the point estimates represents the 80% confidence interval.

The distribution of the fragility is modeled as an asymmetric distribution based on the judgement of the sub-team. This asymmetry is modeled using two different normal distributions above and below the median. The standard deviation (cov*median) is scaled by 1.2 for values above the median and by 0.8 for values below the median. This results in a distribution that is skewed to the right (skewed to higher numbers of failures).

The levee fragility group estimates of the ranges of numbers of failures for each zone is based on the total number of failures for each zone. That is, the standard deviation does not apply to a single levee, but rather to the total number of levees in each zone. This impacts the use of the standard deviation in the probabilistic evaluation. Specifically, the distribution is applied to the median number of breaks in each zone (summation of the median number of breaks for each levee in a zone). This distribution is truncated at 1.5 standard deviations above and below the median.

The coefficients for these models are listed in Table C-1.

Fragility Curves for Non-Liquefaction Induced Failures

The median fragility for non-liquefaction induced failures is modeled by a bilinear model:

If

$$\ln(pga)+c_1+c_2M+c_3M^2+c_4M^3 \leq -2.3,$$

then

$$frag_{Ni}(pga,M)=\exp\{p_1+p_2[\ln(pga)+c_1+c_2M+c_3M^2+c_4M^3]+c_{5i}\}$$

otherwise,

$$frag_{Ni}(pga,M)=\exp\{p_1+p_2[\ln(pga)+c_1+c_2M+c_3M^2+c_4M^3]+c_{5i}+p_3\ln(pga)\}$$

The coefficient of variation is modeled by

$$cov_{Ni}=b_{1i}/1.3$$

The factor of 1.3 represents the interpretation that the range on the fragility given in the point estimates represents the 80% confidence interval. A normal distribution is used for the number of failures. This distribution is truncated at 1.5 standard deviations above or below the median.

The coefficients for these models are listed in Table C-2. All of the coefficients are constant for all zones except for C_5 and b_1 which can vary by zone as shown in Table C-2.

C2.   PROBABILISTIC METHODOLOGY

The levee failure probability is an extension of standard probabilistic seismic hazard analysis. The difference is that instead of calculating the probability of the ground motion exceeding a specified value at a location, we compute the probability of specified number of levee failures being exceeded in a single earthquake. That is, we consider the entire levee system simultaneously.

In the following probabilistic seismic hazard analysis, we consider all possible earthquake magnitudes, locations, and ground motion. For each possible earthquake, we then compute the probability of one or more levee failures occurring within the Delta. This process is repeated for two or more failures, three or more failures, and so on.

Let μ_{Lij} be the median number of failures due to liquefaction for the jth levee in the ith zone. Then

$$\mu_{Lij} = frag_{Li}(pga, M) * L_j$$

where $frag_{Li}$ is the median fragility, pga is the median peak acceleration at the center of the island, M is the magnitude of the earthquake, and L_j is the length of the jth levee in miles. The median number of failures for the ith zone is given by:

$$\mu_{Li} = \sum_{j=1}^{Ni} \mu_{Lij}$$

and the standard deviation of the number of failures due to the uncertainty in the ground motion is given by:

$$\sigma_{GLij} = \mu_{Lij} P2 \sigma_{pga}(M)$$

based on propagation of errors. Assuming that the peak acceleration variability is uncorrectable between levees (which is reasonable for separation distance of greater than 500m), then the standard deviation of the total number of failures within the zone is given by:

$$\sigma_{GLi} = \sqrt{\sum_{j=1}^{Ni} \sigma_{GLij}^2}$$

Since the standard deviation due to uncertainty in the fragility is for the zone and not for individual levees, the fragility uncertainty is fully correlated for each levee within a zone. Therefore, the standard deviation of the total number of failures within a zone due to fragility variability is given by:

$$\sigma_{FLi} = \sum_{j=1}^{Ni} \text{cov}_L \, \mu_{Lij}$$

Similar equations are developed for the non-liquefaction induced failures.

We then use a Monte Carlo approach to sample the distributions for the number of failures in each zone and sum the number of failures from liquefaction and non-liquefaction failures for each zone. Finally, we sum up the number of failures for all the zones to get the total number of failures in the levee system. The frequency of failures in the Monte Carlo sampling defines the conditional probability of the number of failures for a given earthquake magnitude and location.

Let (P(fail>N$_F$ I M, A, W, Hx, Hy) be this conditional probability of the number of failures exceeding N for the given magnitude (M), rupture area (A), rupture width (W), energy center along strike (Hx), and energy center along dip (Hy).

Then the rate of failures is given by:

$$v(Fail > N) = \sum_{k=1}^{NF} N_k \int_M \int_A \int_W \int_x \int_y f_m(M) f_A(M) F_W(M) f_x(x) f_y(y) P\left(fail > N_F \mid M, A, W, x, y\right) dM dA dW dx dy$$

where f_m, f_A, f_W, f_x, f_y are the probability density functions for magnitude, rupture area, rupture width, and energy center. The N_k is the rate of earthquake above the minimum magnitude (here taken as 5.0) for the kth source and NF is the number of faults.

In this equation, the conditional probability of failure is multiplied by the probability of the specified earthquake occurring (given that an earthquake has happened) and then multiplied by the rate of earthquake for the given seismic source. This rate of failure is then summed over all the seismic sources to give the total rate of various numbers of levees failing in a single earthquake. A Poisson assumption for the earthquake occurrence is used to convert the rate of failures into a probability of failures. The result is a hazard curve for the number of levee failures in a single earthquake.

Table C-1.
Fragility Model Coefficients for Liquefaction Induced Failures

Coefficient	All Zones	I	II	III	IV
p1	7.33				
p2	3.02				
c1	-3.47				
c2	0.97				
c3	-0.0838				
c4	0.0031				
c5		0.0	-1.55	-2.23	-2.23
b1	0.94				
b2	-2.05				

Table C-2.
Fragility Model Coefficients for Liquefaction Induced Failures

Coefficient	All Zones	I	II	III	IV
p1	-1.32				
p2	0.54				
p3	2.49				
c1	-75.7				
c2	28.6				
c3	-3.61				
c4	0.156				
c5		0.0	-0.115	-0.810	-2.08
b1		0.38	0.38	0.60	0.60